# Closed-form and Numerical Reverberation and Propagation: Inclusion of Convergence Effects

Chris Harrison
Centre for Marine Research and Experimentation (CMRE)
Viale San Bartolomeo 400
19126 La Spezia
Italy
phone: (+39) 0187-527373 fax: (+39) 0187-527331 email: harrison@cmre.nato.int

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https://mailbox.nurc.nato.int

## LONG-TERM GOALS

The flux formulation of propagation has been used to calculate reverberation, target echo, and signal excess very efficiently in a model developed by Harrison called Artemis. This model is used in the operational planning aid MSTPA at CMRE. Propagation in this formulation falls off monotonically with range. The goal of the ONR-funded work is to improve the propagation accuracy by including convergence and focusing effects without compromising the simplicity and efficiency of the approach.

## OBJECTIVES

The objective has been to write out the theory, i.e. start with the modulus-square of the coherent mode sum and reject rapidly oscillating terms to leave fluctuations on a scale of a ray cycle distance. These formulations were to be evaluated in Matlab and compared with each other and with runs of other well-established models, in this case the normal mode model Orca.

## APPROACH

Because the flux method is exactly equivalent to an incoherent mode sum with a high mode density (i.e. treated as a mode continuum), one can start instead with the modulus-square of the coherent mode sum but retain some of the cross-terms instead of rejecting them all, as in the incoherent sum. It can be shown that the ray cycle distance is related to the difference between adjacent mode eigenvalues, so retaining just these terms adds a ray convergence peak structure to the otherwise monotonic decay. This is the basis of the theory, but to obtain any insight one needs to manipulate the solution into a suitable form. In a similar manner one can write down and evaluate formulas for explicit depth averages and running range averages, rather than the implicit average of the usual flux approach.

The model ARTEMIS produces target echo and reverberation over an entire area in a few seconds. It handles arbitrary bathymetry and stratified SSPs by regarding the solution as a sum over a continuum of WKB modes. It is straightforward to combine the above convergence peak term numerically with the existing incoherent sum term. The benefit over calculating a straight mode sum is that modal phase

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differences are handled analytically and only the amplitude of the WKB modes (not their oscillations) is considered. Thus wavenumber sampling can be quite crude and computation times still short.

**WORK COMPLETED**

The theory, comparisons, and findings have been published in a CMRE report and submitted to JASA (see below). This work corresponds to Subtask 1 in the proposal.

**RESULTS**

The mathematics has been successfully developed into three efficient approaches for calculating one-way propagation each of which has been implemented in Matlab, and all have been favourably compared with each other in several environments (e.g. shallow water surface duct and the deep water Munk profile) and also with the wave model Orca run by Peter Nielsen at CMRE. Some examples are shown for a surface duct in Figs 1-4 below. All derivations and comparisons are contained in a published CMRE report. Also a reduced version has been submitted to JASA.

The approach is to take the acoustic intensity to be the modulus-square of the coherent mode sum. When multiplied out this reveals the incoherent sum added to a double sum of some cosine terms. Even after including the WKB mode shapes these are still exact cosines. Rejecting the most rapid interference the arguments all depend on modal differences which can be Taylor expanded, and so to first order the cosines can be summed in closed form. This leaves a single summation which is converted into a continuous angle integral in exactly the same way as the incoherent sum was. The three solutions correspond to (a) this approach with a parameter $N$ being the number of cosine terms analytically summed (related to the true number of modes), (b) a local range average with width $p$ that is easily interpreted in terms of rays, (c) a local depth average with width $q$. For similar amounts of smoothing there are relationships between the three parameters $N$, $p$, $q$. In the comparisons with Orca (which itself is smoothed, or not, in the report) there are also relations between these parameters and the true number of modes.

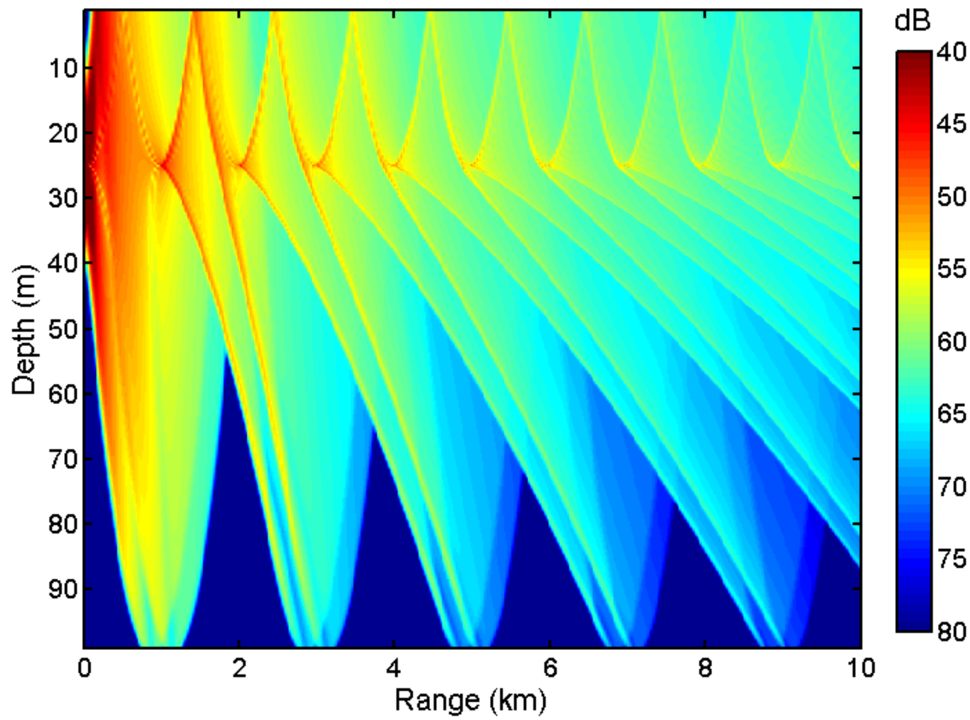


*Fig1: Analytical cosine sum (with N = 20)*

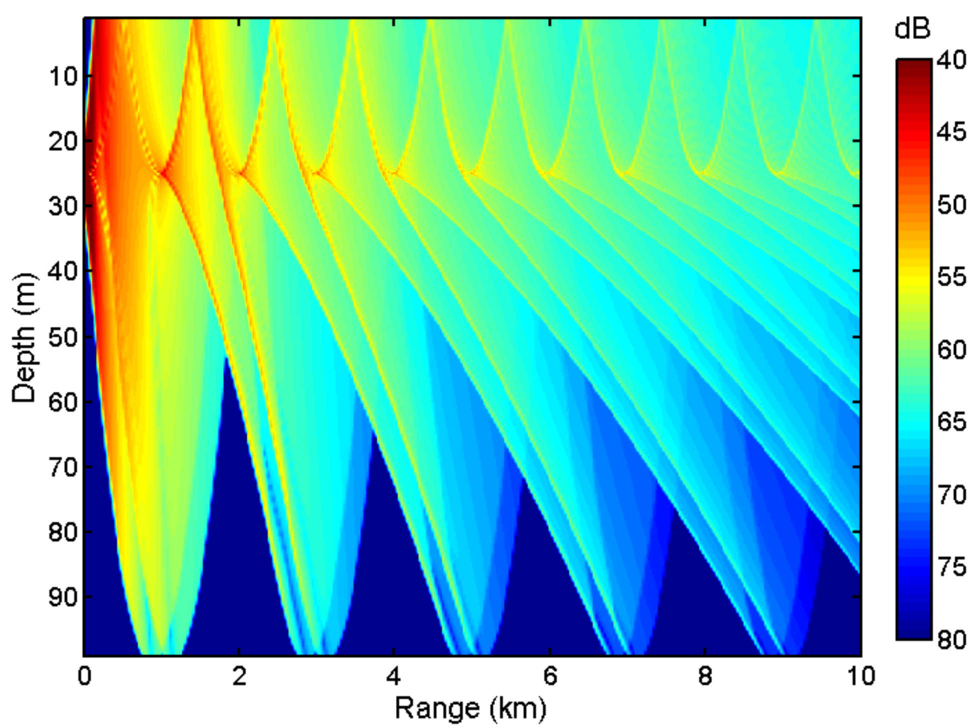


*Fig 2: Local range average (with p = 15)*

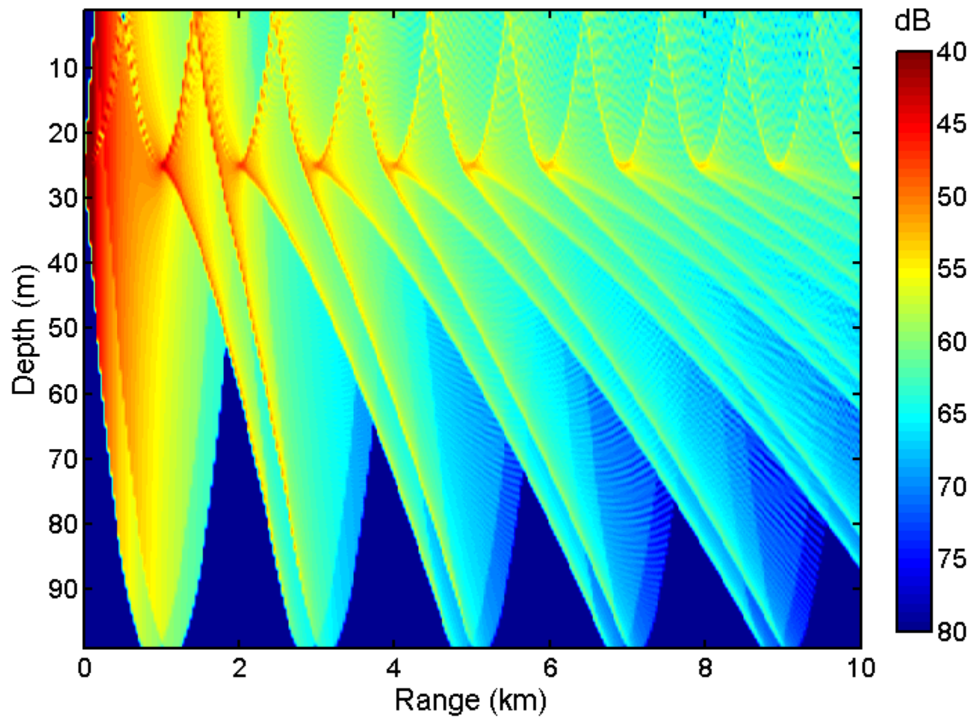


***Fig 3: Local depth average (with q = 0.5)***

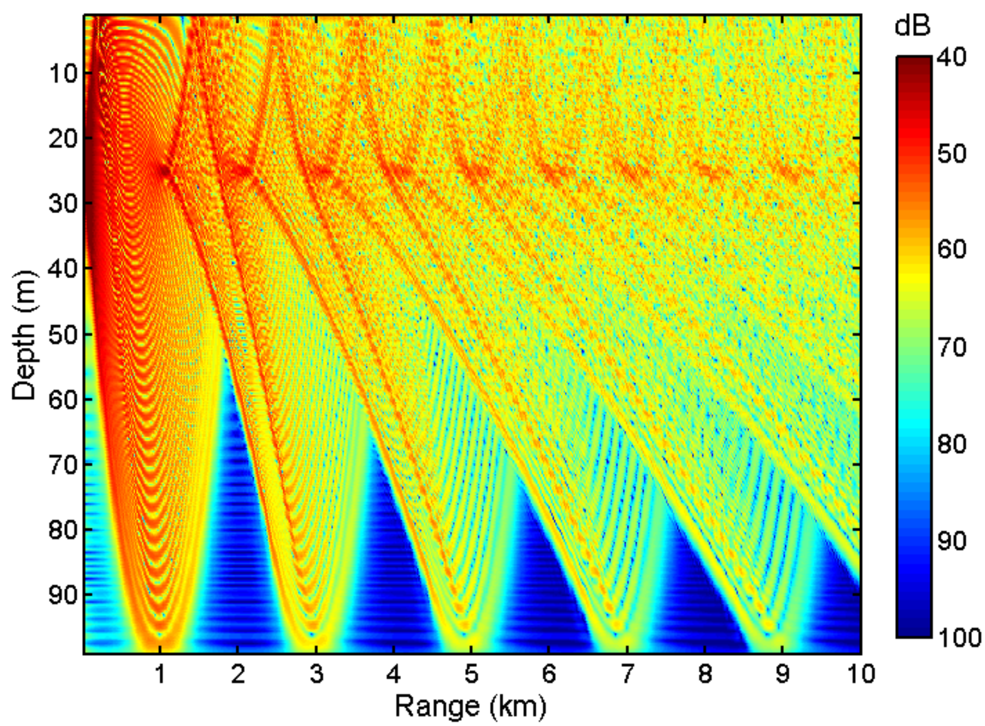


***Fig 4: Equivalent for the wave model Orca at 10kHz***

## IMPACT/APPLICATIONS

The potential and existing Naval application is in fast operational sonar models, operational assessment, sonar assessment, tactical decision aids, operational planning aids. The work so far has found three separate ways of calculating propagation including convergence but excluding rapid modal interference (an analytical mode sum, a range average, and a depth average) that do not compropmise computational speed. Any of these can be incorporated in the sonar assessment model Artemis and this new version can be incorporated in the planning model MSTPA. Choice of which of the three versions, implications for reverberation and range dependence, etc will be the subject of the next part of this work.

## PUBLICATIONS


1. CH Harrison, “Retrieving ray convergence in a flux-like formulstion”, CMRE-FR-2012-006, NATO Unclass, October 2012. [published, refereed]

2. CH Harrison, “Ray convergence in a flux-like propagation formulation”, [submitted to J. Acoust. Soc. Am. November 2012 – MS#12-12549].